1. Suppose the following are $N = 10$ samples of losses on a portfolio:
$\{0.0015, 2.5892, -0.1884, 2.7155, 0.0002, -4.2795, 0.1135, 0.0926, -3.6226, 0.2430\}$.
Set $\alpha = 0.75$.

(a) Compute the $VaR_\alpha$

(b) Compute the $ES_\alpha$ for the above loss sequence.

2. The losses of a security can assume only three values: -0.05, 0, 0.05. In a very long series of independent samples of that losses we observe the following frequencies, shown in Table 1.

<table>
<thead>
<tr>
<th>Values</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.05</td>
<td>50%</td>
</tr>
<tr>
<td>0</td>
<td>47%</td>
</tr>
<tr>
<td>0.05</td>
<td>3%</td>
</tr>
</tbody>
</table>

Table 1: Losses Table.

(a) Compute the empirical CDF of losses.

(b) In the same setting of the previous part, compute the historical VaR at the following confidence levels:
$\alpha = 90\%, \alpha = 95\%, \alpha = 97\%, \alpha = 99\%$.

3. Suppose the tails of losses of a portfolio follow a Pareto distribution:

$$P(L \geq x) = \left(\frac{x}{x_{min}}\right)^k, x_{min} > 0, k < -1$$

(a) Compute the $VaR$ at $\alpha$ confidence level.

(b) Compute the $ES$ for the above losses.

4. Suppose the pdf of given losses is exponential:

$$f(x) = \begin{cases} 
0, & x < 0 \\
\lambda e^{-\lambda x}, & \text{otherwise}
\end{cases}$$

(a) Compute the $VaR$ at $\alpha$ confidence level.

(b) Compute the $ES$ for the above losses.
5. Suppose the loss of a portfolio is normally distributed with mean $\mu$ and variance $\sigma^2$. Fix $\alpha \in (0, 1)$. Compute the Expected Shortfall at confidence level $\alpha$ ($ES_{\alpha}$) of the portfolio.

6. Suppose that the change of the asset value $X$, over one period, follows the standard $t$-distribution with $\nu$ degrees of freedom. Compute the $100(1 - \alpha)\%$ expected shortfall (ES) of a long position over one period. The density of a standard $t$-distribution with $\nu$ degrees of freedom is

$$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$