

504 Question 1 – AMS QUALIFYING EXAM – January, 2006

Let $a_1 \geq a_2 \geq \cdots \geq a_k \geq 0$. Compute

$$\lim_{n \rightarrow \infty} (a_1^n + a_2^n + \cdots + a_k^n)^{1/n}.$$

504 Question 3 – AMS QUALIFYING EXAM – January, 2006

Let E_n , $n = 1, 2, \dots$ be dense open sets in \mathbb{R}^k . Prove that $\bigcap_{n=1}^{\infty} E_n$ is dense in \mathbb{R}^k .

504 Question 2 – AMS QUALIFYING EXAM – January, 2006

For $n = 0, 1, 2, \dots$, and $x \in \mathbb{R}$, define $P_n(x)$ by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

Prove that P_n has n distinct zeros in $(-1, 1)$.