

## Problems from AMS 504 for Qualify Exam

1. Given that  $\sum a_n$  be a convergent series. Prove that  $\sum a_n^2$  also converges absolutely.
2. Suppose  $f$  is a differentiable function with  $f'(x) \geq c > 1$  for all  $x \in \mathbb{R}$ . (a) Prove that  $f(x) - x$  has no upper bound. (b) Does  $f$  have a fixed point? Justify your answer. (c) (Contraction Mapping Theorem) Let  $I = [a, b]$  be a closed real interval.  $g : I \rightarrow I$  is a contraction mapping with contraction constant  $k \in (0, 1)$ , which means, for any  $s_1, s_2 \in I$ ,  $|g(s_1) - g(s_2)| \leq k|s_1 - s_2|$ . Then  $g$  has a unique fixed point  $z \in I$ , such that  $g(z) = z$ .