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504 Question 1 - AMS QUALIFYING EXAM - January, 2009

If

$$C_0 + \frac{C_1}{2} + \cdots + \frac{C_{n-1}}{n} + \frac{C_n}{n+1} = 0$$

where  $C_0, \dots, C_n$  are real constants, prove that the equation

$$C_0 + C_1x + \cdots + C_{n-1}x^{n-1} + C_nx^n = 0$$

has at least one real root between 0 and 1.

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**504 Question 2** - AMS QUALIFYING EXAM - January, 2009

Prove that  $f$  is Riemann-Stieltjes integrable with respect to  $\alpha$  over  $[a, b]$ ,  $f \in \mathcal{R}(\alpha)$ , if and only if for every  $\varepsilon > 0$  there exist a partition  $P$  such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$$