

Spring 2010 Common Quals – CAM, AMS

504.

(1) If A is a countable set and B is an uncountable set, prove that $B \setminus A$ has the same cardinal number as B .

(2) Let f be a continuous real function on R^1 with the following properties: $0 \leq f(t) \leq 1$, $f(t+2) = f(t)$ for every t . And $f(t) = 0$ (for $0 \leq t \leq \frac{1}{3}$), $f(t) = 1$ (for $\frac{2}{3} \leq t \leq 1$). Put $\phi(t) = (x(t), y(t))$ where $x(t) = \sum_{n=1}^{\infty} 2^{-n} f(3^{2n-1}t)$, $y(t) = \sum_{n=1}^{\infty} 2^{-n} f(3^{2n}t)$. Prove that $\phi(t)$ is continuous.