AMS 505, qual (Spring, 2007)

1. Suppose that A and B are two $n \times n$ positive definite matrices. Prove $det(A + B) > 2\sqrt{det(AB)}$.

2. Let A be a $m \times n$ real matrix and B be a $n \times m$ real matrix, with $m \geq n$. Prove that $det(\lambda I_m - AB) = \lambda^{m-n} det(\lambda I_n - BA)$.