

## AMS-505

### Qualifying Problems

- (1).
1. In two dimensional cartesian space, find the linear operators which rotate a vector  $45^\circ$  counterclock-wise and  $30^\circ$  clock-wise, respectively.
  2. Find  $\sin(15^\circ)$  in algebraic form.
  3. Find the projector  $P$  which project a vector onto the line making a  $15^\circ$  counterclock-wise angle with the  $x$ -axis.
  4. Show that the projector  $P$  satisfies  $P^n = P$  and  $(I - P)^n = I - P$ .
  5. What operator is  $I - P$ ?

- (2). The spectral radius of an  $n \times n$  matrix  $A$  is defined as the maximum magnitude of eigenvalues of  $A$ , that is

$$\rho(A) = |\lambda_i|_{max}$$

where  $\lambda_i$  is an eigenvalue of  $A$ . If  $\rho(A) < 1$ , show that

1.  $\lim_{n \rightarrow \infty} A^n = 0$ .

2.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} A^i$$

is convergent.

3. The iteration  $x_{k+1} = Ax_k + b$  is convergent as  $k \rightarrow \infty$ .

4.

$$A + A^3 + A^5 + A^7 + \cdots = A(I - A^2)^{-1}$$