1. Let A and B be $n \times n$ matrices with elements, which are real numbers.Suppose that the eigenvalues of A and B are distinct. Prove that $AB = BA$ if and only if they have the same eigenvectors.

2. Let A and B be $n \times n$ matrices of real numbers, where A is symmetric and B is skew-symmetric, i.e. $A = A^T$ and $B = -B^T$. Suppose further that $AB - BA = I$, where I is the $n \times n$ identity matrix . Prove that unless $x = 0$,

$$|| Ax || \, . \, || Bx || \geq \frac{1}{2}.$$

Hint: $|| x ||^2 = x^T I x$.