

AMS 505, qual (Summer, 2007)

1. Suppose that A is a $n \times n$ matrix which commutes with any nonsingular matrix of order n . Prove $A = kI_n$, where k is a scalar and I_n is an identity matrix.

2. Prove that for any $n \times n$ matrix A has the decomposition $A = QR$, where Q is a $n \times n$ orthogonal matrix and R is a $n \times n$ nonsingular upper triangular matrix.