

AMS505 problems for January 2009 common qualifying exam.

1. Derive the upper and lower bounds on the relative error  $\frac{\|\delta x\|}{\|x\|}$  of the solution to the linear system  $Ax = b$  caused by the perturbation of the right hand side  $\delta b$ . Show that the error bounds remain unchanged if the Cholesky decomposition of the matrix  $A$  is performed (assume that  $A$  is positive definite),  $A = (L\sqrt{D})(\sqrt{D}L^T)$ , and the solution is found via two solves of the corresponding triangular systems.

2. Find the minimum value of  $R(x) = \frac{x_1^2 + x_1x_2 + x_2^2}{2x_1^2 + 2x_1x_2 + x_2^2}$ .