

**AMS Qualifying Examination, January 2009**  
**Probability Questions**

1. Bob tossed a symmetric coin  $(n+1)$  times independently and counted the number of tails. John did the same  $n$  times. Compute the probability that Bob counted more tails than John.
2. Let  $(X, Y)$  be a bivariate random variable, where  $X$  is an exponential random variable with mean 1. Suppose that  $\text{Cov}(X, Y) = -2$ ,  $E[Y] = -2$ , and  $\text{Var}(Y) = 4$ . Find the cumulative distribution function of  $Y$ .
3. Let  $X_1, X_2, X_3$  be three independent uniformly distributed random variables on  $[0, 1]$ . Compute  $P\{X_3 > X_1 + X_2\}$ .
4. Let  $X$  be a random variable with  $E[X] = \mu$  and  $E(X - \mu)^4 = b$ . Is it true that  $P\{|X - \mu| \geq t\} \leq b/t^4$ ? Prove your answer.