AMS Common Exam - Part A, June 2011

Name: _____ ID Num. _____

Section 1: _____ / 75

Section 2: _____ / 75 Total: _____ / 150

This component of the exam (Part A) consists of two sections (Linear Algebra and Advanced Calculus) with four problems in each. Each question is worth 25 points; choose **THREE** questions to answer from **EACH** section. Each problem should be solvable in approximately 20 minutes or less. Provide your answer in the space provided, and show all work. If extra sheets are used, place them inside the booklet and note on the cover page how many additional pages are included.

Good Luck!

Section 1: Linear Algebra

Choose three of the four problems to solve.

1. Let $A, B \in R^{n \times n}$ be two linear operators

   (a). Assume $A = \frac{1}{2}(B + I_n)$, prove $A^2 = A$ if and only if $B^2 = I_n$.

   (b). What types of linear operators are A and B, respectively?

   (c). Given a two dimensional vector $(x, y)^T = (2, 3)^T$, project the vector onto the line $x = y$.

   (d). Reflect the vector in (c) about the line $x = y$.

2. Consider the system

$$\begin{aligned} x + 2y + z &= 3 \\ ay + 5z &= 10 \\ 2x + 7y + az &= b \end{aligned}$$

(a). Find those values of a for which the system has a unique solution.

(b). Find those pairs of values (a, b) for which the system has more than one solution.

(c). Find the basis of the solution space in (b).

3. Consider the following subspaces of R^5:

$$U = span(u_1, u_2, u_3) = span\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$$

$$W = span(w_1, w_2, w_3) = span\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$$

Find a basis and the dimension of $U \cap W$.

4. Let G be the linear operator on R^3 defined by

$$G(x, y, z) = (2y + z, x - 4y, 3x)$$

(a). Find the matrix representation of G relative to the basis

$$S = \{w_1, w_2, w_3\} = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$$

(b). Verify that $[G][v] = [G(v)]$ for any vector v in R^3.

Section 2: Advanced Calculus

Choose three of the four problems to solve.

1. Evaluate the following limits:

   (a).

   $$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$$

   (b).

   $$\lim_{x\to+\infty}\left(\frac{2}{\pi}\arctan x\right)^{x}$$

2. Prove the following equations:

(a).

$$\arcsin x + \arccos x = \frac{\pi}{2},\ x \in [0, 1],$$

(b).

$$3\arccos x - \arccos(3x - 4x^3) = \pi,\ x \in [-\frac{1}{2}, \frac{1}{2}].$$

3. Show that

$$\int_0^{\pi} \frac{x\mathrm{d}x}{1+\cos^2 x} = \frac{\pi^2}{2\sqrt{2}}.$$

4. Compute the surface integral

$$I = \int_{\Sigma} (x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}) \cdot d\mathbf{S},$$

where Σ is the surface of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$