

# AMS Qualifying Exam (January 2011): Probability Questions

Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

**problems to be graded:**

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**Name (PRINT CLEARLY), ID number**

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1. Let  $X_1, X_2, \dots$  be a sequence of independent and identically distributed random variables with common cumulative distribution function  $F$  satisfying  $F(x) < 1$  for all  $x$ . For a given constant  $x$ , define  $Y(x) = \min\{i : X_i > x\}$ . Find  $P(Y(x) > E[Y(x)])$ . What is the limit of this probability as  $x \rightarrow \infty$ ?
2. An airport bus has 7 stops on its route. Assume that there are 25 (distinct) passengers on the bus and all passengers are equally likely to get off at any stop and that all passengers behave independently. The bus stops only if someone wants to get off. Find the expected number of stops the bus will make.
3. Let  $X, Y, Z$  be independent exponential random variables with respective parameters  $\lambda, \mu, \nu$ . Find the probability  $P(X < Y < Z)$ .
4. A factory has produced  $n$  products, each one is defective with probability  $p$  independent of everything else. A test is applied to detect whether a product is defective. For each product, if a product is not defective then the test indicates this. Otherwise, if a defect is present, the test will detect the defect with probability  $q$ . Let  $X$  denote the number of defective products and  $Y$  be the number of products detected as defective. Find the conditional expectation  $E[X|Y]$ .