

Part B: Probability Common Qualifying Exam Questions for June 2006

DO 3 out of 4

1. Independent trials are performed. If the  $i$ th such trial results in a success with probability  $P_i$ , compute the expected number and variance of the number of successes that occur in the first  $n$  trials.
2. Type  $i$  light bulbs function for a random amount of time having a distribution with mean  $\mu_i$  and standard deviation  $\sigma_i$ ,  $i = 1, 2$ . A light bulb randomly chosen from a bin of bulbs is a type 1 bulb with probability  $p$ , and a type 2 bulb with probability  $1-p$ . Let  $X$  denote the lifetime of the bulb. Find  $E(X)$  and  $\text{var}(X)$ .
3. Let  $X_1, X_2, X_3$ , and  $X_4$  be independent continuous random variables having a common distribution function  $F$  and density function  $f$ , and let  $I = P\{X_1 < X_2 < X_3 < X_4\}$ . Compute  $I$ .
4. The random variable  $X$  is stochastically larger than  $Y$ , written  $X \geq_{st} Y$ , if for all  $t$ ,  $P\{X > t\} \geq P\{Y > t\}$ . Show that if  $X \geq_{st} Y$ , where  $X$  and  $Y$  are nonnegative random variables, then  $E[X] \geq E[Y]$ .