

Part B Probability Questions for Common Qualifying Examination, Spring 2007

1. A fair coin is independently flipped  $n$  times,  $k$  times by Mary and  $n-k$  times by Bill. Find the probability that Mary and Bill flip the same number of heads.

2. Let  $X$  be a random variable that takes on values between 0 and  $c$ . That is,  $P\{0 \leq X \leq c\} = 1$ . Show that

$$\text{var}(X) \leq \frac{c^2}{4}.$$

3. If the density of  $X$  equals  $Ce^{-2x}$ , if  $x > 0$ , and 0, if  $x < 0$ . Find  $P\{X > 2\}$ .

4. The random variable  $X$  is the length of time from the start of a course of medication until the remission of illness symptoms recorded in weeks. There are two types of patients: fast responders (who are 90% of patients) and slow responders. Given that a patient is a fast responder, the distribution of  $X$  is exponential with mean 2 weeks. Given that a patient is a slow responder, the distribution of  $X$  is exponential with mean 50 weeks. Find the (unconditional) mean and variance of  $X$ .