

AMS Qualifying Exam (June 2011): Probability Questions

Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded:

Name (PRINT CLEARLY), ID number

1. Let X and Y be discrete random variables with joint mass function

$$f(x, y) = \frac{C}{(x + y - 1)(x + y)(x + y + 1)}, \quad x, y = 1, 2, 3, \dots$$

Find the marginal mass function of X and compute C .

2. A coin shows heads with probability p . Let Y_n be the number of tosses required to obtain a run of n consecutive heads. Find $E[Y_n]$.

3. Let X and Y be two continuous random variables with joint density function

$$f(x, y) = \frac{1}{2}(x + y)e^{-(x+y)}, \quad x, y \geq 0.$$

Find the density function of $Z = X + Y$.

4. A six-sided *fair* die is independently thrown four times. Let M be the smallest of the four numbers obtained, and let S be the sum of the largest three numbers obtained. Find $E[M]$ and $E[S]$.