DOCTORAL QUALIFYING EXAMINATION

FALL 2000

Advanced Calculus & Linear Algebra

NAME: ______________________________

ID#: ______________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: September 5, 2000

Time of Exam: 1:30PM-3:30PM

Place of Exam: Social Behavioral Science Bldg., Room S228

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1. Let S be the set of rational numbers in $[0,1]$ consisting of $0, 1$, and other rational numbers of the form (p/q), where q 2^n, n a positive integer, and p is odd.

   (a) What are the limit points of S?

   (b) Is S closed? Why or why not?

ID # ____ ____ ____

2. Find the volume of the solid enclosed by intersecting right circular cylinders $x^2 + y^2 = 4$ and $y^2 + z^2 = 4$.

ID # ____ ____ ____ ____

3. Let $f(p, v, t) = 0$, where f is a continuous function of independent real variables (p, v, t) and has continuous partial derivatives. Let $f(p_0, v_0, t_0) = 0$. Moreover, the first-order partial derivatives f_p, f_v, f_t do not vanish at (p_0, v_0, t_0). As a consequence of Implicit Function Theorem, in a neighborhood of (p_0, v_0, t_0), $p = p(v,t), v = v(p,t), t = t(p,v)$ satisfy the equation

$$f(p(v,t), v(p,t), t(p,v)) = 0.$$

Prove the following identities:

(a)

$$\frac{\partial p}{\partial t}|_v \frac{\partial t}{\partial v}|_p = -\frac{\partial p}{\partial v}|_t.$$

(b)

$$\frac{\partial p}{\partial t}|_v \frac{\partial t}{\partial v}|_p \frac{\partial v}{\partial p}|_t + 1 = 0.$$

The subscript indicates the variable treated as a constant in partial differentiation.

ID # ___ ___ ___ ___ _

4. Determine whether the following system of linear equations has a unique solution, no solution or a multitude of solutions. Prove your assertion.

$$
\begin{array}{rcrcrcrcl}
x_1 & + & 3x_2 & + & 7x_3 & - & x_4 & = & 0 \\
x_1 & - & 6x_2 & - & 8x_3 & + & 3x_4 & = & 1 \\
3x_1 & + & x_2 & + & x_3 & - & 4x_4 & = & 3 \\
x_1 & - & x_2 & + & x_3 & + & x_4 & = & 4
\end{array}
$$

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5. Let A and B be two similar $n \times n$ matrices. Prove the following:

(a) A and B have the same eigenvalues.

(b) If A is nonsingular, that is, A^{-1} exists, then

$$\left(A^T\right)^{-1} = \left(A^{-1}\right)^T.$$

(A^T denotes the corresponding transpose matrix.)

ID # ___ ___ ___ ___

6. An ellipse with center (6,2) is represented by the equation

$$25\,x^2 + 20\,xy + 40\,y^2 - 340\,x - 280\,y + 1264 = 0.$$

By a suitable change of variables, reduce the above equation to the canonical form

$$a\,x^2 + b\,y^2 = 1.$$