

DOCTORAL QUALIFYING EXAMINATION

FALL 2000

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME: _____

ID#: _____

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: September 5, 2000

Time of Exam: 3:30PM-4:30PM

Place of Exam: Social Behavioral Science Bldg., Room S228

2. Let L denote the vector space of $n \times n$ matrices with real coefficients. For two elements A and B , define $\langle A, B \rangle = \text{tr}(AB^*)$, where B^* denotes the transpose of B .
- (i) Show that $\langle \cdot, \cdot \rangle$ defines an inner product (= dot product) on L .
 - (ii) Let S be the subspace of L consisting of symmetric matrices. Determine the orthogonal complement of S and prove your assertion.

4. A real-valued function f defined on the real line \mathbb{R} is said to be **periodic** if there exists a number $k > 0$ such that $f(x + k) = f(x)$ for all $x \in \mathbb{R}$. Suppose that $f: \mathbb{R} \rightarrow \mathbb{R}$ is continuous and periodic. Prove that f is bounded and uniformly continuous on \mathbb{R} .