Doctoral Qualifying Examination

Spring 2000

Advanced Calculus & Linear Algebra

Name: _____

ID #: _____

Start your answer on each question sheet. Attach all extra sheets you may have used to the appropriate sheet. Hand in all question sheets.

Time: 9:00AM-11:00AM
Date: January 19, 2000
Place: SBS N101

ID #: _____

1. Let $f(x)$ be a real-valued differentiable function with $f'(x) = 0$ for all real values of x. Prove that $f(x)$ is a constant.

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2. Prove that for $\sigma > 0$,

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \exp(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2)dx = 1.$$

ID #: _____

3. Prove that for all values of $x > 0$,

$$\left(1 + \frac{1}{x}\right)^x < 2.9 \quad .$$

ID #: _____

4. Consider the matrix mapping $A : R^4 \rightarrow R^3$ where

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & -1 & 2 & -1 \\ 1 & -3 & 2 & -2 \end{pmatrix}.$$

Find a basis and the dimension of (a) the image of A and (b) the kernel of A.

ID #: _____

5. Let A be a $n \times n$ real symmetric matrix. Prove that all its eigenvalues are real and the eigenvectors corresponding to different eigenvalues are orthogonal.

ID #: _____

6. For the quadratic form $Q(x, y, z) = 3x^2 + 2xy + 3y^2 + 2xz + 2yz + 3z^2$

   (a) Find a symmetric matrix A such that $(A\mathbf{u}, \mathbf{u})$ is equal to $Q(x, y, z)$, where $\mathbf{u} = (x, y, z)^T$.

   (b) Using an orthogonal change of coordinates diagonalize $Q(x, y, z)$.