DOCTORAL QUALIFYING EXAMINATION

FALL 2001

Advanced Calculus & Linear Algebra

NAME: ____

ID#: ____

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: AUGUST 27, 2001

Time of Exam: 1-3PM

Place of Exam: **Stony Brook Union, Room 231**

ID#: ______________________

1. Evaluate $I = \int_0^1 dx \int_x^1 e^{-y^2}\, dy$.

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2. Let $x_0 = 0$, $x_1 = 1$, $x_n = \frac{1}{2}(x_{n-1} + x_{n-2})$ $(n = 2, 3, \cdots)$. Find $\lim_{n\to\infty} x_n$.

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3. Find the absolute maximum and minimum values of $f(x, y) = x^2 + 12xy + 2y^2$ defined on the domain $\{(x, y) : 4x^2 + y^2 \leq 25\}$.

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4. Find the determinant of $\begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} & x_2^{n-1} \\ \vdots & & & & & \\ 1 & x_{n-1} & x_{n-1}^2 & \cdots & x_{n-1}^{n-2} & x_{n-1}^{n-1} \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} & x_n^{n-1} \end{pmatrix}$.

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5. Suppose A is an n-th order anti-symmetric matrix, i.e. $A^T = -A$. Let $D = diag\{d_1, d_2, \cdots, d_n\}$ be a diagonal matrix whose entries $d_1, d_2, \cdots, d_n$ are greater than zero. Prove $det(D + A) > 0$.

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6. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix over real field R. Find necessary and sufficient conditions on a, b, c, and d so that A is diagonalizable.