DOCTORAL QUALIFYING EXAMINATION

FALL 2001

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME : ______________________________

ID# : ______________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: AUGUST 27, 2001

Time of Exam: 3-4PM

Place of Exam: **Stony Brook Union, Room 231**

ID#: ______________________

1. Let $\mathbf{V}$ and $\mathbf{W}$ be vector spaces, and let $T : \mathbf{V} \to \mathbf{W}$ be a linear transformation. If $\mathbf{V}$ is finite-dimensional, prove that

$$\text{nullity}\,(T) + \text{rank}\,(T) = \dim\,(\mathbf{V}).$$

ID#: ____________________

2. Let A and B be $n \times n$ matrices.

(a) Prove that AB and BA have the same eigenvalues.
(b) Prove or disprove that AB and BA are similar matrices.
(c) If λ is an eigenvalue of an invertible matrix A, prove that λ^{-1} is an eigenvalue of A^{-1}, the inverse of A.

ID#: ______________________

3. If X is a metric space, if $E \subset X$, and if E' denotes the set of all limit points of E in X, then the *closure* of E is the set $\overline{E} = E \cup E'$. Prove that $\overline{E}$ is the smallest closed set that contains every point of E.

ID#: ______________________

4. Prove that $\sum_{n=0}^{\infty} \frac{1}{n!} < 3$ and deduce from the inequality that the series converges.