

Doctoral Qualifying Examination

Spring 2001

Advanced Calculus and Linear Algebra

Name: _____

ID# _____

Start your answer on each question sheet. Attach all extra sheets you may have used to the appropriate sheet. Hand in all questions sheets.

Time: 1-3PM

Date: January 24, 2001

Place: SB Union Rm 231

ID #: _____

1. Prove that the shortest distance from the origin to the curve of intersection of the surfaces $xy = a$ and $y = bx$ where $a > 0$, $b > 0$, is $3\sqrt{a(b^2 + 1)}/2b$.

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2. Evaluate $\int \int \int_R \sqrt{x^2 + y^2 + z^2} dx dy dz$, where R is the region bounded by the plane $z = 3$ and the cone $z = \sqrt{x^2 + y^2}$.

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- Determine the number of positive real roots for the equation $x^a = a^x$, where the constant $a > 0$.

ID #: _____

4. Suppose the $n \times n$ matrix A commutes with every nonsingular $n \times n$ matrix. Show that $A = kI_n$ for some scalar k , where I_n is the n -th order identity matrix.

ID #: _____

5. Suppose U and W are subspaces of V for which $U \cup W$ is also a subspace. Show that either $U \subset W$ or $W \subset U$.

6. Let $A = \begin{pmatrix} 1 & -1 & -1 \\ 3 & -4 & -2 \\ 2 & -3 & -2 \end{pmatrix}$.

(a) Find the LU factorization of A .

(b) Let X_k denote the solution of $AX = B_k$. Find X_1, X_2, X_3, X_4 when $B_1 = (1, 1, 1)^T$ and $B_{k+1} = B_k + X_k$ for $k > 0$.