Doctoral Qualifying Examination

Spring 2001

Linear Algebra & Analysis

Name:________________________________

ID#__________________________________

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers: . Hand in all questions sheets.

1 2 3 4

Time 3-4 PM
Date: January 24, 2001
Place: SB Union Rm 231

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1. Let A be a square matrix with real entries. Recall that A^T denotes the transpose of A, that A is **skew-symmetric** if $A^T = -A$, that A is **orthogonal** if $A^T = A^{-1}$ (equivalently: $A^T A = I$), and that for a real parameter t, $exp(tA)$ is defined as the (always convergent) power series:

$$exp(tA) = I + tA + \frac{t^2A^2}{2!} + \ldots = \sum_{i=0}^{i=\infty} \frac{t^iA^i}{i!}$$

(a.) Show that if A is skew-symmetric then $exp(tA)$ is orthogonal for all values t.

(b.) Show that the converse of part (a.) is also true.

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2. Let A and B be $n \times n$ matrices with complex coefficients. Suppose that AB has n distinct eigenvalues $\{\lambda_1, \ldots, \lambda_n\}$. Show that BA has the same eigenvalues as AB.
Note: You are asked to prove a special case of a general theorem, to which you **cannot** appeal: AB and BA have the same eigenvalues, same multiplicities, and same characteristic function.

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3. Suppose that $f(x)$ is real-valued, continuous, and decreasing on $[0, \infty)$, and $f(n) \to 0$ as $n \to \infty$. Define $\{a_n\}$ by

$$a_n = f(0) + f(1) + \ldots\ldots + f(n-1) - \int_0^n f(x)dx .$$

(a) Prove directly from the definition that $\{a_n\}$ is a Cauchy sequence.

(b) Evaluate $\lim_{n \to \infty} a_n$ if $f(x) = e^{-x}$.

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4. Prove that every uncountable set of real numbers has a limit point.