DOCTORAL QUALIFYING EXAMINATION

FALL 2002

Advanced Calculus & Linear Algebra

NAME____

SOLAR ID#____

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Note: Answer all questions.

Date. September 3, 2002

Time of Exam 9AM-11AM

Place of Exam: Stony Brook Union Auditorium

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1\.
Given:

$$\int_0^\infty \frac{\sin x\,dx}{x} = \frac{\pi}{2}.$$

Derive the value of

$$\int_0^\infty \frac{\sin^2 x\,dx}{x^2}\,.$$

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2.

Let $f(x) = 3x^2$ for $0 \leq x \leq 1$ and $f(x) = 4 - x$ for $1 \leq x \leq 4$. Let R be the region bound by the x-axis, the graph of f and the straight line segments $x = b$ and $x = b + 2$ connecting the graph to the x-axis. Find the value of b for which the area of R is maximum.

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3. Prove by mathematical induction the identity

$$\sum_{k=0}^{n} C(n,k) = 2^n.$$

Here $C(n,k)$ denote the binomial coefficients $\frac{n!}{k!(n-k)!}$.

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4. Let $T : R^3 \to R^3$ be the linear transformation $T(v) = Av$, where

$$A = \begin{bmatrix} 5 & 5 & 2 \\ -6 & -6 & -5 \\ 6 & 6 & 5 \end{bmatrix}$$

Find **ker**(T).

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5.

Let A be a skew-symmetric $n \times n$–real matrix, i.e., $A^T = -A$.

(a) Show that A is singular if n is odd.

(b) Show that all the eigenvalues of A are purely imaginary

Solar ID#______________________________

6.

Let $T : R^2 \rightarrow R^2$ be an affine transformation (a mapping which maps straight line segments into straight line segments) defined by the relation $T(u) = Au + b$, where b is a specified vector in the cartesian plane and A is a 2×2-matrix :

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}.$$

A vector v is called a *fixed point* of T if $Tv = v$. Show that T has a unique fixed point if $(p-1)(s-1) - qr \neq 0$.