DOCTORAL QUALIFYING EXAMINATION

SPRING 2002

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME : ____

ID# : ____

Start your answer on each question sheet. Attach all extra - the appropriate sheet. Hand in all question

Date: JANUARY 23, 2002

Time of Exam: 3-4PM

Place of Exam: **Physics Building, Room P-116**

ID#: ______________________

1. Let T be an invertible linear operator on a finite-dimensional vector space V.

   (a) Show that for any eigenvalue λ of T, λ^{-1} is an eigenvalue of T^{-1}.

   (b) Prove that the eigenspace of T corresponding to λ is the same as the eigenspace of T^{-1} corresponding to λ^{-1}.

   (c) Prove that if T is diagonalizable, then T^{-1} is diagonalizable.

ID#: ______________

2. Let T be a linear operator on a finite-dimensional inner product space V.

(a) Show that $N(T^*T) = N(T)$, where $N(T^*T)$ is the null space of T^*T and $N(T)$ is the null space of T.

(b) Show that $\text{rank}(T^*T) = \text{rank}(T)$, $\text{rank}(T) = \text{rank}(T^*)$, and $\text{rank}(TT^*) = \text{rank}(T)$.

(c) Show that if T is normal, then $N(T^*) = N(T)$ and $R(T^*) = R(T)$, where $R(T)$ represents the range of T and $R(T^*)$ represents the range of T^*.

ID#: ____________________

3. Prove that the series $\sum_{n=1}^{\infty} \frac{x}{n(x+n)}$ defines a continuous function on $[0, \infty)$.

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4. Let G be an open cover of a set $A \subset R^1$. Prove that there exists a countable subfamily of G that also covers A.