

DOCTORAL QUALIFYING EXAMINATION

FALL 2003

Advanced Calculus & Linear Algebra

NAME: _____

ID#: _____

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: September 3, 2003

Time of Exam: 10-12noon

Doctoral Qualifying Examination
Fall 2003
Advanced Calculus and Linear Algebra

Note: Answer all questions.

1. Prove the following inequalities:

(a) For $0 < x < 1$,

$$0 < \ln(1+x) < \arctan x < \arcsin x < \frac{\pi}{2}.$$

(b)

$$\frac{\pi}{4} + \frac{3}{25} < \arctan\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}.$$

2. Evaluate the integral

$$\int_0^1 2\sqrt{(2x+1)} dx.$$

(The radical is the power of 2!)

3. The perimeter of a right-angled triangle is known to be 100 centimeters. Calculate its hypotenuse if the area of the triangle is maximum possible.

4. If a matrix T is nonsingular, $T^{-1}AT$ is called a **similarity transformation**.
- (a) Show that if A and B are reduced to diagonal matrices under the same similarity transformation, then A and B commute, i.e. $AB = BA$.
- (b) Show that no pair of 2×2 matrices A and B satisfy the relation $AB - BA = I$, where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

5. Let A and B be two nonsingular $n \times n$ matrices with real elements. Prove that AB and BA have the same eigenvalues.

6. Find an orthogonal transformation which reduces the quadratic form xy to the diagonal form $c_1x^2 + c_2y^2$.