DOCTORAL QUALIFYING EXAMINATION

FALL 2003

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME: ______________________________

SOLAR ID#: ______________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Time of Exam: 12noon-3PM

1. Prove that if the real-valued function f is uniformly continuous on a finite interval I, then f is bounded on I.

2. Prove that $|\sin x - \sin y| \leq |x - y|$.

3. Let A be a Hermitian matrix.

(a) Show that all the eigenvalues of A are real.

(b) For all nonzero vectors x, show that

$$\lambda_1 \leq \frac{x^* A x}{x^* x} < \lambda_n,$$

where λ_1 is the smallest eigenvalue of A, and λ_n is the largest eigenvalue of A. x^* is the conjugate transpose of x.

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4. Let A and B be $m \times n$ and $n \times p$ matrices over $\mathbb{R}$. Here null(A) is the dimension of the null space of the matrix A:

(a) Prove that

$$\text{null}(AB) \leq \text{null}(A) + \text{null}(B);$$

(b) Prove that

$$\text{rank}\,(A) + \text{rank}\,(B) \leq \text{rank}\,(AB) + n.$$