DOCTORAL QUALIFYING EXAMINATION

SPRING 2003

Advanced Calculus & Linear Algebra

NAME: ______________________________

ID#: ______________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: JANUARY 22, 2003

Time of Exam: 11-1PM

Place of Exam: **Stony Brook Union Auditorium**

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1. (a) Show that the vectors (2,1,-1), (1,0,1) and (0,1,-1) form a basis for R^3 - the three-dimensional Euclidean Space.

   (b) Express (1,0,0),(0,1,0),and (0,0,1) in terms of the above basis.

   (c) What are the matrices which represent this change of bases?

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2. Find the eigenvalues and eigenvectors (or generalized eigenvectors) of the 3×3 matrix which has zeros on the principal diagonal and all other entries are 1. Write the vectors so that they form an orthogonal set.

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3. Let $u_1 = 2$. Define

$$u_{n+1} = \frac{1}{2}\left(u_n + \frac{3}{u_n}\right), n = 1, 2, 3, \ldots \quad .$$

(a) Prove that the sequence $\{u_n\}$ converges.

(b) Find the limit as $n \to \infty$.

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4. (a) Let m be a nonnegative integer and $a > -1$. Show that

$$\int_0^1 x^a (\ln x)^m \, dx = \frac{(-1)^m \, m!}{(a+1)^{m+1}}.$$

(b) Evaluate

$$\int_0^1 \frac{x \, dx}{\sqrt{1-x^2}\,(x+2)(x^2+1)}.$$

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5. (a) Prove that for $0 \leq x \leq \frac{\pi}{2}$,

$$\frac{2}{\pi} \leq \frac{\sin x}{x} \leq 1.$$

(b) Show that for $x > 0$ all the zeros of the function,

$$f(x) = cx - \tan x,$$

are simple zeros, i.e. $f(x)$ and $f'(x)$ can not be zero at the same point.

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6. Let A be the real symmetric $3\times3-$matrix described in Problem 2. Express A as $Q' D Q$, where D is a diagonal matrix, Q' is the transpose of Q, where Q is an orthogonal matrix.