DOCTORAL QUALIFYING EXAMINATION

SPRING 2003

Probability

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME: ______________________________

SOLAR ID#: ______________________________

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: January 23, 2003

Time of Exam: 1-2PM

Place of Exam: Stony Brook Union Auditorium

ID#: ______________________

1. Each day an experimental animal is exposed to a certain set of stimuli designed to elicit a particular response. Let A_k be the event that the animal makes the desired response on the kth day, A_k' be the event that the animal does not make the desired response on the kth day, and suppose that $P(A_{k+1} \mid A_k) = \beta$ and $P(A_{k+1} \mid A_k') = \alpha$, where $0 < \alpha < \beta \leq 1$. Let $p_k = P(A_k) = 1 - P(A_k')$.
   a. Show that $p_{k+1} = \alpha + (\beta - \alpha)p_k$.
   b. If $\beta = 1$ and $p_1 = 0$, show that $p_k = 1 - (1-\alpha)^{k-1}$.
   c. Show that $\lim_{k\to\infty}(p_k) = \dfrac{\alpha}{1+\alpha-\beta}$.

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2. A tea-drinking lady starts out with two boxes, each of which contains n teabags. Each time she needs a cup of tea she selects one of the two boxes at random and takes a teabag from it. What is the probability that the $(n+k)$th teabag will empty one of the boxes, where $0 \le k < n$?

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3. Let X, Y, and Z be independent random variables all of which have the exponential density $f(x) = e^{-x}, x > 0$. Find the probability of the simultaneous occurrence of the events that $X \leq Y$ and $X \leq 2Z$.

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4. Let X and Y be independent random variables with means both zero and common variance σ^2. Let $Z = 2X + Y$. Find $E(Z^2 \mid X = x)$.