DOCTORAL QUALIFYING EXAMINATION

SPRING 2003

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

NAME: _____

SOLAR ID#: _____

Start your answer on each question sheet. Attach all extra sheets you use to the appropriate sheet. Hand in all question sheets.

Date: January 23, 2003

Time of Exam: 1-2PM

Place of Exam: Stony Brook Union Auditorium

ID#:_____

1. Let A be an $n \times n$ skew-symmetric matrix (*i.e.*, $A^t = -A$) with entries from $\mathbb{R}$. Prove the following:
   (a) If n is odd, then A is not invertible.
   (b) No matter what n is, $I - A$ is invertible.
   (c) No matter what n is, $(I - S)^{-1}(I + S)$ is an orthogonal matrix.

ID#:_____

2. Let S and T be commuting linear operators on a finite dimensional vector space V over $\mathbb{C}$.
   (a) Let W be an eigenspace of S. Show that W is invariant under T.
   (b) Prove that S and T have a common eigenvector.
   (c) Assume that V has a basis $\beta = \{e_1, e_2, \cdots, e_n\}$ which is composed of eigenvectors of T, *i.e.*, $Te_i = \lambda_i e_i$ for $1 \leq i \leq n$, and the eigenvalues are distinct, *i.e.*, $\lambda_i \neq \lambda_j$ for $i \neq j$. Then show that S is diagonal in the same basis $\beta = \{e_1, e_2, \cdots, e_n\}$.

ID#: _____

3. Suppose that a sequence $\{f_n\}$ of functions converges pointwise on an interval $[a, b]$ and, for each x in $[a, b]$, there is an open interval I_x containing x such that $\{f_n\}$ converges uniformly on $I_x \cap [a,b]$. Prove that $\{f_n\}$ converges uniformly in $[a,b]$.

ID#: _____

4. Let $f(x) = 0$ if x is irrational and $f(p/q) = 1/q$ if p and q are positive integers with no common factors. At what points is f continuous? At what other points is f discontinuous? Prove your claims.