Common Exam, Part B

Spring 05 CAM QUESTIONS ID#________________

(1). Prove that a $n \times n$ matrix,which is not the Identity Matrix I_n, can not have 1 as a repeated eigenvalue with the same algebraic and geometric multiplicity n.

(2). Let W denote the subspace of R^5 defined by the property that $u \epsilon W$ if and only if its coordinates in the standard basis sum to zero. Find the orthogonal projection of (1,-1,1,0,0) in W.

(3). Let $f : \mathbb{R} \to \mathbb{R}$ be continuously differentiable and such that $\lim_{x\to\infty} f(x)$ exists and is finite. Prove or disprove by counterexample: $\lim_{x\to\infty} f'(x) = 0$.

(4). Prove the Contraction Mapping Theorem: If X is a complete metric space, and if $g : X \to X$ is a contraction, then there exists a unique $x \in X$ such that $g(x) = x$.