Doctoral Qualifying Examination

Fall 1999

Linear Algebra & Analysis

Solve any three problems for a full credit. Indicate below EXACTLY which three problems you have attempted by circling the appropriate numbers:

1 2 3 4

Name: _____

ID#: _____

Start your answer on each question sheet. Attach all extra sheets you may have used to the appropriate sheet. Hand in all question sheets.

Time: 10:30 – 11:30 AM, September 1, 1999
Place: SBS N106

ID #: _____

1. Let V be the vector space of $n \times n$ matrices over the field of real numbers. Let T be a linear transformation on V which maps a matrix into its transpose.

   1. Find the eigenvalues of T.

   2. Describe the eigenvectors corresponding to each eigenvalue of T.

   3. Suppose $n = 2$. Find an ordered basis for V such that the matrix representation of T in this basis is diagonal.

ID #: _____

2. Let T be a linear operator on a finite-dimensional real inner product space V. Then T is self-adjoint if and only if there exists an orthonormal basis for V consisting of eigenvectors of T. Use this result to prove the following:

V has an orthonormal basis of eigenvectors of T with corresponding eigenvalues of absolute value 1 if and only if T is self-adjoint and unitary.

ID #: _____

3. Let f be a real-valued function having a finite derivative f' on (0, 1]. Assume that $|f'(x)| < M$, $x \in (0, 1]$, for some constant M. Prove that

$$\lim_{n\to\infty} f(\frac{1}{n})$$

exists.

ID #: _____

4. Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.