

**Applied Mathematics and Statistics  
Common Qualifying Examination Part B  
in Computational Applied Mathematics**

**Summer 2011 (May)**

**(Closed Book Exam)**

**Please solve 3 out of 4 problems for full credit.**

Indicate below which problems you have attempted by circling the appropriate numbers:

**Part B:**                    1                    2                    3                    4

**NAME \_\_\_\_\_**

Start each answer on its corresponding question page. Print your name, and the appropriate question number at the top of any extra pages used to answer any question. Hand in all answer pages.

Date of Exam: May 31st, 2011

Time: 11:00 AM – 12:00 PM

**B1.** Find the general solution of the following Riccati equation

$$y' + 2xy = 1 + x^2 + y^2,$$

given that  $y = x$  is a solution.

**B2.** Classify the type of singular point at 0 and  $\infty$  of the differential equation  $\tan(x)y' = y$ .

**B3.** Let  $\mathbf{A} \in \mathbb{R}^{m \times n}$  be a matrix with full rank, where  $m \geq n$ . Let  $\mathbf{R}^T \mathbf{R}$  be the Cholesky factorization of  $\mathbf{A}^T \mathbf{A}$ .

- a) Show that  $\mathbf{A} \mathbf{R}^{-1}$  is composed orthonormal column vectors.
- b) Show that  $\mathbf{A}$  has the same nonzero singular values and corresponding right singular vectors as  $\mathbf{R}$ .

**B4.** In the conjugate gradient method for solving  $\mathbf{Ax} = \mathbf{b}$ , show that the subspace spanned by the first  $m$  search directions is the same as the Krylov subspace  $\mathcal{K}_m = \langle \mathbf{b}, \mathbf{A}\mathbf{b}, \mathbf{A}^2\mathbf{b}, \dots, \mathbf{A}^{m-1}\mathbf{b} \rangle$ .