

Fall 1999

NAME: _____

The four problems you have attempted: _____

Instructions: Work four of the following six problems. You may not use notes or any other assistance.

1. Let X_n converge to c in probability. If $h(x)$ is a continuous function at $x = c$, prove that $h(X_n)$ converges to $h(c)$ in probability.

2. Let \bar{X}_n denote the mean of a random sample of size n from a gamma distribution with pdf

$$f(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}, \quad x > 0, \alpha > 0.$$

Find the limiting distribution of $\sqrt{n}(\bar{X}_n - \alpha)/\sqrt{\bar{X}_n}$.

3. Suppose that X_1, \dots, X_n is a random sample from the exponential distribution with pdf

$$f(x|\theta) = \theta e^{-\theta x}, \quad x > 0, \theta > 0.$$

Find the UMVUE of $e^{-k\theta}$ for fixed $k > 0$.

Hint: Define

$$U = \begin{cases} 1, & X_1 > k \\ 0, & X_1 \leq k. \end{cases}$$

4. Let Y_n be the n th order statistic of a random sample of size n from a distribution with pdf $f(x|\theta) = 1/\theta$, $0 < x < \theta$, zero elsewhere. Suppose the prior distribution on θ has pdf $\lambda(\theta) = \beta\alpha^\beta/\theta^{\beta+1}$, $\theta > \alpha$, zero elsewhere, with $\alpha > 0$ and $\beta > 0$. Find the Bayes estimator of θ under the squared error loss.

5. Let the distribution of X be given by

x	0	1	2	3
$P_\theta(X = x)$	θ	2θ	$0.9 - 2\theta$	$0.1 - \theta$

where $0 < \theta < 0.1$. For testing $H_0 : \theta = 0.05$ against $H_1 : \theta > 0.05$ at level $\alpha = 0.05$, determine if the following test is UMP:

$$\phi(1) = 0.5, \phi(0) = \phi(2) = \phi(3) = 0.$$

6. Find the likelihood ratio test of $H_0 : a = 1$ versus $H_1 : a \neq 1$ based on a sample X_1, \dots, X_n from a $N(\theta, a\theta)$ family, where θ is unknown.