

Spring 2000

NAME: _____

Instructions: Work three of the following four problems.

1. A child is missing in an amusement park, and it is presumed that she is equally likely to be in any of 4 possible areas. Let γ_i denote the probability that the child will be found upon a search of the i th area when she is, in fact, in that area, $i = 1, 2, 3, 4$. What is the conditional probability that the child is in the i th area, given that a search of area 1 is unsuccessful, $i = 1, 2, 3, 4$?
2. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student would get 4 or more correct answers just by guessing?
3. Let X be a random variable that takes on values between 0 and a . That is, $P(0 \leq X \leq a) = 1$. Show that

$$\text{Var}(X) \leq a^2/4.$$

Hint: Show

$$E(X^2) \leq aE(X)$$

and then use this to show that

$$\text{Var}(X) \leq a^2[\beta(1 - \beta)],$$

where $\beta = E(X)/a$.

4. If X_1, X_2, \dots, X_n are independent and identically distributed random variables having uniform distributions over $[0, 1]$, find
 - (a) $E[\max(X_1, \dots, X_n)]$;
 - (b) $E[\min(X_1, \dots, X_n)]$.