

Fall 2001

NAME: _____

Instruction: Work three of the following four problems.

1. An urn contains n balls, numbered 1 through n . If m balls are randomly withdrawn in sequence, each time replacing the ball selected previously, find $P(X = k)$, $k = 1, \dots, m$, where X is the maximum of the m chosen numbers.
2. A standard Cauchy random variable has density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty.$$

If X is a standard Cauchy random variable, show that $1/X$ is also a standard Cauchy random variable.

3. The joint density of X and Y is given by

$$f(x, y) = C(y - x)e^{-y}, \quad -y < x < y, \quad 0 < y < \infty.$$

Find C .

4. The twelve players on a basketball team consist of 3 centers, 4 forwards, and 5 backcourt players. If the players are paired up at random into four groups of size 3 each, find the expected number of triplets consisting of one of each type of player.