PROBABILITY THEORY QUALIFYING EXAMINATION

Spring 2001

NAME: _____

Instructions: Work three of the following four problems.

1. Let V be the random variable representing time to onset of the tumor of interest and W the time to death from this tumor. Suppose V is distributed as Uniform$(0, a)$, for a positive constant a and given $V = v$, W is distributed as Uniform(v, a). Define $X = W - V$.

   (a) Find the pdf of X.

   (b) Find the expected value $\mathrm{E}(X)$ of the elapsed time between the tumor onset time and the time until death from the tumor.

2. In relation to continuous nonnegative random variables T that might represent lifetimes or times to failure, with pdf $f(t)$ and cdf $F(t)$, we define the failure rate function as

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(t < T \leq t + \delta t | T > t).$$

   (a) For the Weibull pdf given by $f(t) = \beta\lambda(\lambda t)^{\beta-1} \exp[-(\lambda t)^{\beta}]$, $t > 0$, with parameters $\lambda > 0$ and $\beta > 0$, obtain the failure rate function $\lambda(t)$, $t > 0$.

   (b) Show that $F(t) = 1 - \exp\left[-\int_0^t \lambda(u)du\right]$ and $f(t) = \lambda(t) \exp\left[-\int_0^t \lambda(u)du\right]$, $t > 0$ in general.

3. An urn contains M white and N black balls. If a random sample of size r is chosen, what is the probability that it contains exactly k white balls?

4. Suppose that $P[X = 0] = 1 - P[X = 1]$. If $E[X] = 3\mathrm{Var}(X)$, find $P[X = 0]$.