

PROBABILITY THEORY
QUALIFYING EXAMINATION

Fall 2002

NAME: _____

Instruction: Work three of the following four problems.

1. There are two local factories that produce radios. Each radio produced at factory A is defective with probability 0.05, whereas each one produced at factory B is defective with probability 0.01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory A or factory B . If the first radio that you check is defective, what is the conditional probability that the other one is also defective?
2. The random variables X , Y and Z are independent and uniformly distributed in the interval $[0, 1]$. Find the PDF (probability density function) of $W = X + Y + Z$.
3. The random variable X has the PDF

$$f_X(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional expectation of X^2 given that $X > 1.5$.

4. Let X be the number of 3's and let Y be the number of 5's in four independent rolls of a 6-sided die. Find the joint PMF (probability mass function) of X and Y .