PROBABILITY THEORY QUALIFYING EXAMINATION

Spring 2002

NAME: ______________________________

Instruction: Work three of the following four problems.

1. Let $\{A_n\}$ be a sequence of sets. Show the Bonferroni's inequality,

$$P(\cap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$

2. Suppose that X takes on one of the values 0, 1, 2. If for some constant c, $P(X=i) = cP(X=i-1)$, $i = 1, 2$, find $\mathrm{E}(X)$.

3. Let X be a standard normal random variable, Find the PDF of the random variable $Y = 2X^2 + 1$.

4. Consider two independent random variables X and Y whose PMFs are given by

$$p_X(x) = \begin{cases} \frac{1}{2} & \text{if } x = 0, 1, \\ 0 & \text{elsewhere,} \end{cases} \qquad p_Y(y) = \begin{cases} \frac{1}{2} & \text{if } y = 1, 2, \\ 0 & \text{elsewhere.} \end{cases}$$

Let R be the random variable that takes with equal probability either the value of X or the value of Y. Let G denote the sum of six independent experimental values of R. Find the mean and variance of G.