

Fall 1999

NAME: \_\_\_\_\_

**Instructions:** Work three of the following four problems.

1. Independent trials, consisting of rolling a pair of fair dice, are performed. What is the probability that an outcome of 5 appears before an outcome of 7, when the outcome of a roll is the sum of the dice?

2. As a simplified model for weather forecasting, suppose that the weather (either wet or dry) tomorrow will be the same as the weather today with probability  $p$ . If the weather is dry on January 1, let  $P_n$  denote the probability that it will be dry  $n$  days later.

(a) Show that

$$\begin{aligned}P_n &= (2p - 1)P_{n-1} + (1 - p), \quad n \geq 1 \\P_0 &= 1.\end{aligned}$$

(b) Prove that

$$P_n = \frac{1}{2} + \frac{1}{2}(2p - 1)^n, \quad n \geq 0.$$

3. A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?

4. Let  $X$  be a nonnegative random variable. Prove that

$$\mathbf{E}(X) \leq [\mathbf{E}(X^2)]^{1/2} \leq [\mathbf{E}(X^3)]^{1/3} \leq \dots.$$