

Qualifying Exam (May 2011): Quantatative Finance

You have 4 hours to do this exam.

Reminder: This exam is closed notes and closed books.

Do 2 out of problems 1,2,3.

Do 5 out of problems 4,5,6,7,8,9,10.

All problems are weighted equally. **On this cover page write which seven problems you want graded.**

problems to be graded:

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination.

Name (PRINT CLEARLY), ID number

Signature

(1).

(2).

(3).

(4). Derive the Black-Scholes equation for a stock S step by step in detail. Mention any assumption you might use. State Itô's lemma and note where and how you apply Itô's lemma (don't need to prove Itô's lemma). In the Black-Scholes world, price a European option with payoff of $\max(S_T^2 - K, 0)$ at time T .

(5). Team A plays team B, in a series of 7 games, whoever wins 4 games first wins. You want to bet \$100 that your team wins the series, in which case you will receive \$200, or \$0 if they lose. However the broker only allows bets on individual games. You can bet \$ X on any individual game the day before it occurs to receive \$ $2X$ if it wins and \$0 if it loses. How do you achieve the desired pay-out? In particular, what do you bet on the first game?

(6). Explain how you can use Monte Carlo simulation to price a European call option (assuming stock price follows a geometric Brownian motion)?

(7). Apply Itô's lemma to 2^{W_t} , where W_t is a Brownian motion, is it a martingale?

(8). You are constructing a simple portfolio using two stocks A and B . Both have the same expected return of 12%. The standard deviation of A 's return is 20% and the standard deviation of B 's return is 30%; the correlation of their return is 50%. How will you allocate your investment between these two stocks to minimize the risk of your portfolio?

(9). Suppose we are doing a random walk on the interval $[0, 1000]$, starting at 80. With probability $1/2$, this number increases or decreases by one at each step. We stop when one of the boundaries (0 or 1000) is reached. What is the probability that this boundary will be 0?

(10). The Average Value-At-Risk is a popular coherent risk measure defined by

$$\text{AVaR}_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_x(X) dx$$

where X is the portfolio return. If the distribution of R is continuous then we have the following property

$$\text{AVaR}_\alpha(X) = -E[X|X < -\text{VaR}_\alpha(X)] = \frac{1}{\alpha} E[-X 1_{\{X < -\text{VaR}_\alpha(X)\}}]$$

where

$$1_{\{X < -\text{VaR}_\alpha(X)\}} = \begin{cases} 1, & \text{if } X < -\text{VaR}_\alpha(X) \\ 0, & \text{if } X \geq -\text{VaR}_\alpha(X) \end{cases} .$$

Find closed form solution of $\text{AVaR}_\alpha(X)$ if X follows Student's t distribution. Recall the density function of Student's t distribution is

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi\nu}} \left(1 + \frac{x^2}{\nu}\right)^{-\left(\frac{1+\nu}{2}\right)}$$

where ν is the degrees of freedom and $\Gamma(\cdot)$ denotes the Gamma function.

Scratch Paper 1

Scratch Paper 2

Scratch Paper 3

Scratch Paper 4

Scratch Paper 5

Scratch Paper 6