Rate of Return on a Stock

• When considering investments, helpful to describe statistical characteristics, ignoring transaction costs
• Stocks tend to have positive expected returns, generally +10% to 20%, depending on stock
• Stocks also have corresponding standard deviations, generally 15% to 30%
• Estimating these quantities is important
Example:

Let’s examine the characteristics of Yahoo (YHOO) and Disney (DIS) between 9/30/09 and 10/26/09.

Average Sample Return

Sample Standard Deviation of the Returns

Covariance, Correlation
Portfolio Mean and Variance

Suppose we have n assets with random rates of return \( r_1, r_2, \ldots, r_n \)

These rates of return have expected values

\[
E(r_1) = \bar{r}_1, \quad E(r_2) = \bar{r}_2, \ldots, \quad E(r_n) = \bar{r}_n
\]

If we form a portfolio of these assets with weights \( w_i, \quad i = 1, 2, \ldots, n \). The rate of return of the portfolio is

\[
r = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n
\]
Mean Return of a Portfolio

If we take the expected value of both sides

\[ E(r) = w_1 E(r_1) + w_2 E(r_2) + \ldots + w_n E(r_n) \]

Expected rate of return of a portfolio is the weighted sum of the expected rates of the return of the individual assets from which the portfolio is composed.
Variance of Portfolio Return

Let $\sigma^2$ be the variance of the portfolio return and the covariance of the return of asset $i$ with asset $j$ by $\sigma_{ij}$.

The variance of a portfolio’s mean can be calculated from the covariances of the pairs of asset returns and the asset weights.

$$\sigma^2 = E[(r - \bar{r})^2] = \sum_{i, j=1}^{n} w_i w_j \sigma_{ij}$$
Two Asset Portfolio Example

We have two assets with covariance .01

Asset 1: \( r_1 = .12, \sigma_1 = .20 \)
Asset 2: \( r_2 = .18, \sigma_2 = .18 \)

Calculate the mean, variance, and standard deviation of a portfolio with weights \( w_1 = .25 \) and \( w_2 = .75 \)
Diversification

• A portfolio with very few assets may be risky, as measured by variance
• Variance of a portfolio can be reduced by adding more assets
• The way portfolio variance is measured implies that assets with low covariance to each other can help lower overall variance
• Extreme case: all assets uncorrelated
Portfolio of Uncorrelated Assets

Suppose we have $n$ assets, equally weighted, uncorrelated, with mean $m$ and variance $\sigma^2$

Return of the portfolio is

$$r = \frac{r_1}{n} + \frac{r_2}{n} + \ldots + \frac{r_n}{n} = \frac{1}{n} \sum_{i=1}^{n} r_i$$

The expected value of this is $m$ and is independent of $n$
Variance of a Portfolio

Variance of the portfolio is

\[ Var(r) = \frac{1}{n^2} \sum_{i=1}^{n} \sigma_i^2 = \frac{\sigma_2}{n} \]

When assets are uncorrelated, portfolio variance can be made arbitrarily small.

When assets are correlated there may be a limit to how much variance can be lowered.
Variance of Correlated Assets

Assume each asset has rate of return with mean $m$ and variance $\sigma^2$, and each return pair has covariance $.3\sigma^2$ for $i \neq j$

The variance of the such a portfolio is

$$Var(r) = E\left[\sum_{i=1}^{n} \frac{1}{n} (r_i - \bar{r})\right]^2 = \frac{.7\sigma^2}{n} + .3\sigma^2$$