Chapter 5 - Applications

• Capital Budgeting
• Bond Portfolio Construction
• Management of Dynamic Investments
• Valuation of Firms from Accounting Data
Capital Budgeting

• What is the best way to spend money?
• Allocation of resources among projects and investments for which there aren’t well established markets and where projects require discrete expenditures of cash
• Defined in terms of scale, cash requirements, benefits
• Budget is a limitation in funding projects – not all projects may be funded so choices must be made
Independent Projects

• Selecting from a list of $m$ potential projects where:
  • $b_i$ is the total benefit of the $i$th project, usually expressed as a net present value
  • $c_i$ is the initial cost
  • $C$ is the total budget
  • Define $x_i$ for each $i = 1,2,...,m$ is zero if the project is rejected and 1 if the project is accepted
Independent Projects Lead to Integer Programming Problem

\[
\text{maximize } \sum_{i=1}^{m} b_i x_i \\
\text{subject to } \sum_{i=1}^{m} c_i x_i \leq C \\
x_i = 0 \text{ or } 1 \text{ for } i = 1, 2, \ldots, m
\]
Solving Integer Programming

• Exact Method: Zero-One Optimization (software available on Matlab, Mathematica, Splus, Excel)

• Approximate Method: Benefit Cost Ratio Ranking – Projects with a high BCR are desirable subject to the ability of the project to use appropriate amounts of the budget

• Linear Programming: A good course to take in a QF program
Approximate Method

• Compute Benefit Cost Ratio: PV of Total Benefit of Project/Outlay for the Project
• Rank the projects by BCR
• Successively add projects subject to the total not exceeding the capital budget
• Entire budget may not be used
Interdependent Projects

• Several different goals, each with more than one possible project. Fixed budget

• Assume $m$ goals and associated with the $i$th goal are $n_i$ possibilities

\[ x_{ij} = 0 \text{ or } 1 \text{ for } i = 1,2,...,m \text{ and } j = 1,2,...,n_i \]

• $x_{ij}$ is 1 if goal $i$ is chosen and implemented by project $j$, else 0
Interdependent Projects

\[
\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n_i} b_{ij} x_{ij} \\

\text{subject to } \sum_{i=1}^{m} \sum_{j=1}^{n_i} c_{ij} x_{ij} \leq C \\

\sum_{j=1}^{n_i} x_{ij} \leq 1 \text{ for } i = 1, 2, ..., m \\
x_{ij} = 0 \text{ or } 1 \text{ for all } i \text{ and } j
Example 5.2

- County Transportation
- 3 independent goals
- Each goal has possible projects
- Total available budget is $5,000,000
- Find the optimal combination of projects; only one project per goal
- Dependent nature of projects can be addressed with the appropriate constraints
Optimal Portfolios

• Construction of a portfolio of financial securities, including projects
• Distinction between Portfolio Optimization which involves only securities
• Use Excel Solver
• Example: Fixed Income Cash Matching Problem – Investing now to meet a known series of cash flows in the future
Cash Matching Problem

• Known sequence of future money obligations

\[ Y = (y_1, y_2, \ldots, y_n) \]

• Design a portfolio now that will, without alteration, provide the necessary cash flow

• If we have \( m \) bonds, the CF stream associated with bond \( j \) is

\[ c_j = (c_{1j}, c_{2j}, \ldots, c_{nj}) \]
Cash Matching Problem

• The price of bond $j$ is denoted $p_j$

• The amount of bond $j$ to be held is

• The problem can be formulated as follows: Find the $x_j$'s so that the cost of the portfolio is minimized while the obligations are met

• Objective function: minimize the total cost of the portfolio

• Constraints: Cash matching constraints
Cash Matching Problem

minimize $\sum_{j=1}^{m} p_j x_j$

subject to $\sum_{j=1}^{m} c_{ij} x_j \geq y_i$ for $i = 1, 2, \ldots, n$

$x_j \geq 0$ for $j = 1, 2, \ldots, m$
Example 5.3

• Cash obligations over a 6 year period: $100, $200, $800, $100, $800, $1,200 (yearly)

• 10 bonds are selected for this purpose, all have face value $100, with different coupon rates and maturities

• What combination of bonds will provide the cash flow at the lowest cost?
Example 5.3

- Utility: Minimize the cost of the portfolio
- Each year provide required cash flow or more
  
  $y_i = \text{required cash flow in year } i$
  
  $p_j = \text{price of } j\text{th bond}$
  
  $x_j = \text{amount of } j\text{th bond}$
  
  $c_{ij} = \text{cash flow in } i\text{th year of } j\text{th bond}$
  
  $j = 1,2,...,10$ and $i = 1,2,...,6$