

AMS410.01.

Quiz-6

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1. An insurance company examines its pool of auto insurance customers and gathers the following information:

- All customers insure at least one car.
- 70% of the customers insure more than one car.
- 20% of the customers insure a sports car.
- Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

- (A) 0.13
- (B) 0.21
- (C) 0.24
- (D) 0.25
- (E) 0.30

2. An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Prob of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

- (A) 0.13
- (B) 0.16
- (C) 0.19
- (D) 0.23
- (E) 0.40

3. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} |x|/10 & \text{for } -2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the expected value of X .

- (A) $\frac{1}{5}$
- (B) $\frac{3}{5}$
- (C) 1
- (D) $\frac{28}{15}$
- (E) $\frac{12}{5}$

4. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x, y) = \frac{x + y}{27} \text{ for } 0 < x < 3 \text{ and } 0 < y < 3.$$

Calculate the probability that the device fails during its first hour of operation.

- (A) 0.04
- (B) 0.41
- (C) 0.44
- (D) 0.59
- (E) 0.96

5. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

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- (D) 0.59
- (E) 0.96

6. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

- (A) 20
- (B) 29
- (C) 41
- (D) 53
- (E) 70

7. The time, T , that a manufacturing system is out of operation has cumulative distribution function

$$T(t) = \begin{cases} 1 - \left(\frac{2}{t}\right)^2 & \text{for } t > 2 \\ 0 & \text{otherwise} \end{cases}$$

The resulting cost to the company is $Y = T^2$.

Determine the density function of Y , for $y > 4$.

- (A) $\frac{4}{y^2}$
- (B) $\frac{8}{y^{3/2}}$
- (C) $\frac{8}{y^3}$
- (D) $\frac{16}{y}$
- (E) $\frac{1024}{y^5}$

8. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period.

Calculate the probability that the participant was a heavy smoker.

- (A) 0.20
- (B) 0.25
- (C) 0.35
- (D) 0.42
- (E) 0.57

9. The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f , where $f(x)$ is proportional to $(10 + x)^{-2}$. Calculate the probability that the lifetime of the machine part is less than 6.

(A) 0.04

(B) 0.15

(C) 0.47

(D) 0.53

(E) 0.94

10. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

- (A) 0.10
- (B) 0.20
- (C) 0.25
- (D) 0.40
- (E) 0.80