AMS 210 - Practice Final Exam

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1. (20 pts total) Given:

$$H = \begin{matrix} 1 & -3 \\ -3 & 1 \end{matrix}, \quad b = \begin{matrix} 4 \\ 2 \end{matrix}$$

a) (2 pts) Solve for $\det(H)$.

b) (3 pts) Solve for H^{-1}.

c) (2 pts) Solve for x in $Hx = b$, using H^{-1}.

d) (4 pts) Find the condition number of H using the sum norm.

e) (4 pts) Give a bound for $|Hy|_s$ assuming $|y|_s = 2$.

f) (5 pts) Determine the eigenvalues and eigenvectors of H.

2. (10 pts total) Given :

Hours Spent Studying	1	2	3	4	5	6
Hours Required to Finish Test	9	7	6	4	2	1

Fit this data with a regression model of the form $y* = qx + r$, provided the following formulas:

$q = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n\sum x_i^2 - (\sum x_i)^2}$, $\quad r = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{n\sum x_i^2 - (\sum x_i)^2}$, $\quad q' = \frac{\sum x_i' y_i}{\sum (x_i')^2}$, $\quad r' = \frac{1}{n}\sum y_i$,
$r = r' - \frac{q\sum x_i}{n}$

3. (10 pts total) We surveyed a group of students eating vanilla, chocolate, and strawberry ice cream about which flavor they would choose next time. Responses yielded the following transition matrix:

		Current Flavor		
		Vanilla	*Chocolate*	*Strawberry*
Next Time	*Vanilla*	$\frac{1}{4}$	$\frac{1}{2}$	0
	Chocolate	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$
	Strawberry	0	0	$\frac{2}{3}$

a) (5 pts) How many rounds does it take on average for a person to switch from strawberry to vanilla?

b) (5 pts) While switching from strawberry to vanilla, how many times, on average, will a person have chocolate ice cream?

4. (10 pts total) The following problem is about cell growth. Suppose there are three types of cells: young, midlife, and old. Midlife cells create one young cell each period. Old cells also create one young cell each period. Every young cell splits into three midlife cells each period. Every midlife cell splits into three old cells each period. Warning: To solve for the growth multiplier and long-term distribution, you will need an understanding of complex numbers.

a) (2 pts) Produce the Leslie matrix representing this type of cell growth.

b) (5 pts) Find the growth multiplier.

c) (3 pts) What is the long-term distribution?

5. (15 pts total) Given:

$$W = \begin{matrix} 1 & 0 & 1 \\ -2 & 1 & 1 \\ 3 & -1 & 0 \end{matrix}$$

a) (5 pts) Find the basis for the range of W.

b) (5 pts) Find the basis for the nullspace of W.

c) (5 pts) Assume there is a matrix N with the same nullspace as W, and $u = [1\,2\,2]^T$ is in the range of N. Find a vector v in the range of N whose third element, v_3, is 4.

6. (15 pts total) Given $Bx = c$:

$$B = \begin{matrix} -1 & 0 \\ 2 & 1 \\ 1 & -3 \end{matrix},\ x = \begin{matrix} x_1 \\ x_2 \end{matrix},\ c = \begin{matrix} 20 \\ -10 \\ 0 \end{matrix}$$

a) (6 pts) Calculate the pseudoinverse, B^+.

b) (3 pts) Use B^+ to find an approximation of x; call it x^* .

c) (3 pts) What c^* does x^* return?

d) (3 pts) How good of an approximation is x^*?

7. (10 pts total) Given:

$$A = \begin{matrix} 1 & 2 \\ -2 & 2 \end{matrix}$$

a) (8 pts) Find an orthogonal matrix Q and upper triangular matrix R such that $A = QR$.

b) (2 pts) Use Q and R to calculate A^{-1}.

8. (10 pts total) Approximate function $f(x) = x^3 - x^2 + 10$ as a linear combination of the first three Legendre polynomials over the interval [-1, 1]: $L_0(x) = 1, L_1(x) = x, L_2(x) = x^2 - \frac{1}{3}$.