Final for AMS-321 for 2003

Starting time: 5:00 12/15/2003 Monday
Ending time: 7:30 12/15/2003
In Math SINC Site (S-235), on-machine exam

(1) Open Book And Own Lecture Notes
(2) Calculators Allowed
(3) Do ANY Two Of The Four Problems
(4) You Must Mark The Two Problems You Attempt;
(5) Will Only Grade The 1st Two If You Don't Mark The Problems You Attempted;
(6) Each Problem Is Worth 17.5 Points As Indicated;
(7) Max Score Is 35 Points;
(8) NO Additional Points for Doing More Than Two Problems.

Problem 1:

Assume the Port Jefferson line city-bound LIRR train starts with 200 people at Port Jeff station each weekday morning. We further assume

(1) There are 10 stations between the staring Port Jeff and the terminal Penn stations (excluding these two stations) labeled as S1, S2, …, S10.
(2) The number of people boarding each of these 10 stations follows 10 different approximate Normal Distributions with bounds given below.
   a) At S1, there are [10, 20] boarding, (i.e., the min is 10 and max is 20 and the most likely is 15);
   b) At S2, there are [20, 30] boarding;
   c) …
   d) At S10, there are [100, 110] boarding;

(3) The number of passengers leaving the train for all 10 stations follows uniform distribution in [1, 10].
(4) The leaving passengers are those who stay the longest, i.e., first-on-first-off. (This assumption simplifies fare calculation.)

Please do the following:

(1) Compute the numbers of passengers who board and leave the train at each one of the 10 stations, i.e., fill in the following table:

Stations	Number Boarding (5.0 Points)	Number Leaving (2.5 Points)
S1		
S2		
S3		
S4		
S5		
S6		
S7		
S8		
S9		
S10		

(2) Compute the number of passengers who arrive at Penn station regardless of starting station (2.5 Points).
(3) Now, we assume the fare for any two adjacent stations is $1 per passenger. For example, $1 for Port Jeff station to S1 station, but $11 to Penn station. Anther example: if one boards at station S2 but leaves at station S7, the fare is $5. Compute the total fare collected for one complete trip using your rider pattern generated in (1) (7.5 Points).

Problem 2:

A section of the streets and avenues in a city form a 2-dimensional grid: 9 avenues: 1st Ave, 2nd, Ave, …, to 9th Ave (running north-south but labeled East to West) and 10 streets 47th, 48th, to 57th Street (running east-west but labeled South to North). Now, assume you live at the northwest corner of 9th Ave and 57th Street and your friend lives at southeast corner of 1st Ave and 47th Street. We have the following assumptions:

(1) All blocks on streets are 200 meter long;
(2) All blocks on avenues are 100 meter long;
(3) All avenues traffic is one-way;
   a) Odd-numbered avenues going north to south;
   b) Even-numbered avenues going south to north;
(4) All streets are bi-directional;
(5) Speeds are as follows
   a) Speed 5th Ave is 1 meter-per-second;
   b) Speed on 4th and 6th Aves is 2 meter-per-second;
   c) Speed on 3rd and 7th Aves is 3 meter-per-second,
   d) 2nd and 8th Aves is 4 meter-per-second,
   e) 1st and 9th Aves is 5 meter-per-second,
   f) All streets are 3.5 meter-per-second.

With the above assumptions, please find the minimal-time paths for going to your friend's place from yours and back. You may have multiple paths, finding one is sufficient. Your solution should include the time in seconds and the path. (10.0 Points for going to your friends and 7.5 Points for coming back.)

Problem 3:

There are infinitely many infinitely long parallel lines on the floor. The distance between any two adjacent parallel lines is one unit. You drop a stick of length one unit, totally randomly, to the floor 10,000 times. Each time, the stick either stays in the middle of any two of the parallel lines without touching any one of them or intersects with one or both of the adjacent lines. During N experiments, the stick intersects the parallel line M times. Perform numerical experiments to do the following:

1) Compute the ratios of N/M for N =1000, 2000, …, 10,000 and make a table for as follows (10 Points)

N	N/M

1,000	
2,000	
3,000	
4,000	
5,000	
6,000	
7,000	
8,000	
9,000	
10,000	

2) Plot a diagram for the above data (2.5 Points)
3) Explain why, from mathematical point of view, N/M should converge to a well-known number. (5.0 Points)

Problem 4:

A toy rocket of fixed mass *m* is pushed straight up at an initial velocity of v_0 from the surface of earth. Assume earth's gravitational constant *g=9.8 m/s^2* and air frictional force on the rocket *kv* where *k* is the frictional constant and *v* is the instantaneous velocity of the rock where *k=0.5*. The mass of the rocket *m=1.0 kg*. Ignore the rocket "launcher" height. Now, please do the following

1) (5.0 Points) If we want to the rocket to reach the maximal height $h_{max}=2000$ meters, please compute the required minimal velocity v_0.
2) (5.0 Points) If we want to the rocket to stay in the sky for 15 seconds (including going up and down), please compute the required minimal velocity v_0.
3) (7.5 Points) If the rocket is launched at the initial velocities as you computed in (1) and (2) above, please compute the velocities of the rocket just before it hits the ground when it returns.