

Final for AMS321 for 2007

Starting: 11:00 12/21/2007 Friday
Ending: 13:30 Same Day
Location: Math SINC Site (S-235)
Format: On-machine exam

- (1) Open Book And Own Lecture Notes
 - (2) Calculators and Computers Allowed
 - (3) Do ANY Three Of The Five Problems
 - (4) Each Problem is Worth 12 Points (but the max is 35)
 - (5) NO Additional Points for Doing More Than Three Problems
 - (6) Good Luck
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Problem 1: A man's front-yard is a perfect 30x40 (m²) flat rectangle and in this yard the man has 15 trees of random heights: tallest 10m and shortest 1m. These trees are randomly distributed on the flat yard. Now, the man wants to put one Christmas light on the top of each tree. Write a program to show which tree he should start and what path he should follow to wire all trees with the shortest wire (of course without returning to the starting tree.) To get credit, you must give (1) coordinates and heights of all 15 trees (2 *credits*); (2) the starting tree coordinates and the shortest-wire path (8 *credits*); (4) the total wire length (2 *credits*).

Problem 2: A cylindrical tank with a hole at the bottom can empty a full tank of water precisely in 1 hour. Please compute the time needed to drain the tank that is only lower half full. Water leaking follows the following

differential equation: $\frac{dV}{dt} = -k\sqrt{h}$ where V is the volume of water in the tank at time t , h is the height from the leaking hole to the water surface (also at time t), and k is a constant related to the size of the leaking hole.

Problem 3: One 100 particles are, initially, placed on the 100 grid points of a 10x10 uniform square. We assume particle i interacts with particle j by a potential $V(r_{ij}) = \frac{1}{r_{ij}^{12}} - \frac{2}{r_{ij}^6}$ where r_{ij} is the distance from particle i to particle j .

Please do the following (1) compute the potential of all particles (except particle “ i ” itself) on particle i with the initial particle positions (*1 credit*); (2) compute the average potential on all 100 particles with initial particle positions (*1 credit*); (3) Now, assume you can move the particles around from the initial positions freely, try to move them so that the average potential is minimal (*10 credits*).

Problem 4: Minimize the following two-variable function

$$f(x, y) = e^{-(x^2+y^2)}(1 + \sin 2\pi x)(1 + \cos(2\pi y + 0.2))$$

where $x \in [-1, 1]$ and $y \in [-1, 1]$

Problem 5: Mr. Fool bought \$10,000 worth of LowTech.com’s stocks on 12/14/2006, precisely one year ago. Assume he has gone through 52 weeks of trading with exactly five trading days each. Therefore, he has gone through 260 trading days. The change rate of Mr. Fool’s stock from the previous day for the entire year follows an approximately Normal Distribution with values in interval $[-2\%, 1\%]$. Please (1) Compute Mr. Fool’s stock value at the end of each of the 260 trading days. Show the results in a table and a graph (*4 credits*); (2) Do the same as in (1) if the stock changes from the previous day in interval $[-1\%, 2\%]$ (*4 credits*); (3) Do the same as in (1) if the stock changes from the previous day in interval $[-1\%, 1\%]$ (*4 credits*).