AMS 321: Computer Projects in Applied Mathematics

Project 1

Assignment Date: Wednesday (09/9/2009)

Collection Date: Wednesday (09/23/2009) 5PM

Grade: **See Individual Problems**

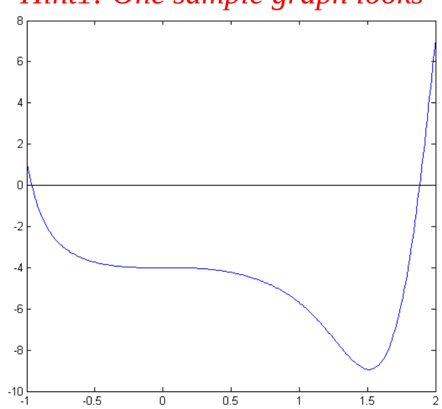
1. **(5 Points)** Write a computer program to solve the following equation (you may use provided math functions such as exp(), sin(), etc)

$$f(x) = 7^{-x^3} - x^2 \sin(x^3) - 1$$

by going through the following steps. (Your results must have 5 digits of accuracy)

(1) Draw a diagram for the function $f(x)$ above in interval $x \in [-1, 2]$ by using Excel or any graphing program of your choice.

Hint1: One sample graph looks



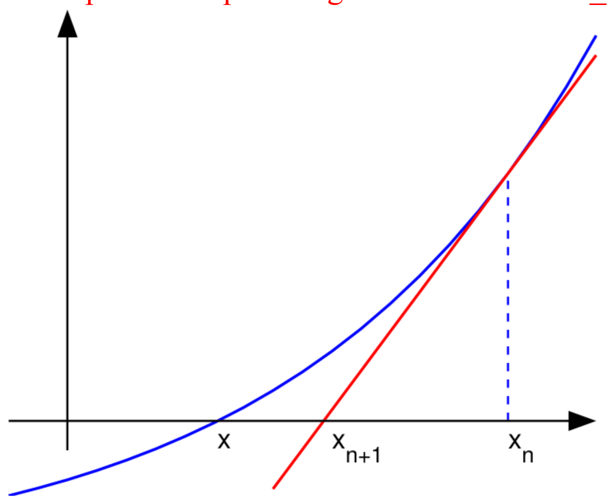
(2) From the graph, please use eyeballs to identify one root (labeled as "x_1"). Report x_1. Naturally, nobody can require you to get accuracy more than 2 digits.

(3) With your rough "root x_1" obtained in (2), create a small interval which contain the root $[x_1 - \epsilon, x_1 + \epsilon]$ where "ϵ" is small enough that within this internal there is only one root. Report " ϵ".

(4) Using bisection method, and the interval you selected in (3), to find the root x_0. Please report the number of iterations you need to get this solution. Report the solution.
Suggested reference "http://bytes.com/forum/thread630544.html"

(5) Next, using Newton's method, and "$x_1 - \epsilon$" as the initial root, to find the root x_0. Please report the number of iterations you need to get this solution. Your method of computing the needed $f'(x)$ term is your choice: you may get the analytical form for $f'(x)$ and then program it or just directly perform numerical derivative on $f(x)$.

*Hint2: View "*http://en.wikipedia.org/wiki/Newton%27s_method"



(6) Do similar work to (5) except that the initial root is now selected as "$x_1 + \epsilon$".

2. **(5 Points)** This problem has three parts:

(1) Iterate $x_{n+1} = \begin{cases} x_n + x_{n-1}, & \text{if } x_n + x_{n-1} < 1 \\ x_n + x_{n-1} - 1, & \text{o.w.} \end{cases}$ to generate 1,000,000 pseudo random floating-point numbers uniformly distributed in interval $(0, 1)$ and then convert them to random numbers uniformly distributed in interval $(-1, 1)$. You don't need to present these numbers, but you need to describe how you generate them. You may start the generation with any two random numbers in internal (0, 1) as x0 and x1.

(2) Count the number of random numbers that are in [-1, -0.9), [-0.9, -0.8), ... [0.9, 1.0] and make a histogram. You may use any method to generate such histogram.

(3) Produce 10,000,000 uniform random numbers by method in (1) and then add every 10 together to form 1,000,000 new numbers. You may use any order to add them up as long as you don't use the same number twice. Count how many new numbers that are in [-10, -9), [-9, -8),..., [9, 10]. Make a histogram for the result.

Hint 3: To put a set $\{x \geq 0\}$ of positive floating-point numbers in B bins, all you need to do it to multiply all $\{x\}$ by a factor λ such that the $\lambda x_{max} = B$ and then a line in the code

$$N[\lambda x]{+}{+};$$

does all the trick.