

# Fun with Graphs

## Problem # 7

The STABLE MARRIAGE problem discussed in class assumes that every person fully ranks people of the opposite sex. Now we wish to consider the case of ties (indifference). We allow a person, say Mary, to rank Peter and Paul as equal, meaning that she is indifferent to which one of the two she would marry.

One can define two types of stable marriages:

*Strong stability:* There does not exist a couple, say Mary and Paul, such that each prefers the other other to their spouse in the matching.

*Weak stability:* There does not exist a couple, say Mary and Paul, such that either

(1) Mary prefers Paul to her spouse and Paul prefers Mary or is indifferent between her and his spouse in the matching,

or

(2) Paul prefers Mary to his spouse and Mary either is indifferent between her spouse and Paul, or prefers Paul to her spouse in the matching.

We note that a matching that is weakly stable is also strongly stable, but not every strongly stable matching is weakly stable.

Does there always exist a strongly stable matching? If so, how would you find one? What about a weakly stable matching? Do you have an example in which no weakly stable matching exists?