# Finite Mathematical Structures A

Exam 1: Thursday, February 25, 2010

READ THESE INSTRUCTIONS CAREFULLY. Do not start the exam until told to do so. Make certain that you have all 5 pages of the exam. You will be held responsible for any missing pages.

Write your answers on this examination, using the backs of pages if needed.

There may be problems that are solvable by inspection, but if you get the wrong answer and have shown no work, then I will assign NO partial credit.

This examination is CLOSED BOOK and CLOSED NOTES. You may not use any books, papers, or materials other than your pen or pencil. You may use a 4 by 6 “cheat sheet”, which should be turned in with your exam.

The following items should NOT be on your desk - turn them off AND put them INSIDE your bag!

- calculator
- cell phone
- pager

If I see any of these items, even turned off, this will be considered cheating!!!

Work carefully, and GOOD LUCK!!!

**Last (Family) Name (PRINT CLEARLY):** ______________________________

**First Name (PRINT CLEARLY):** ______________________________

**ID Number:** ______________________________

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that any suspected instance of academic dishonesty will be reported to the Academic Judiciary and that I will be subjected to the maximum possible penalty permitted under University guidelines.

**Signature:**

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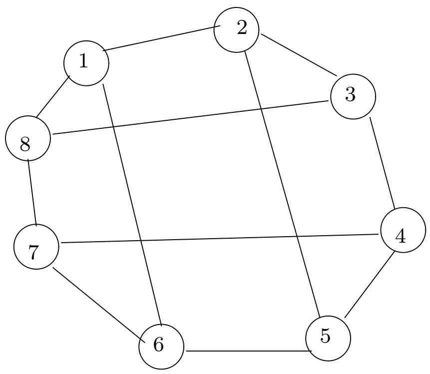
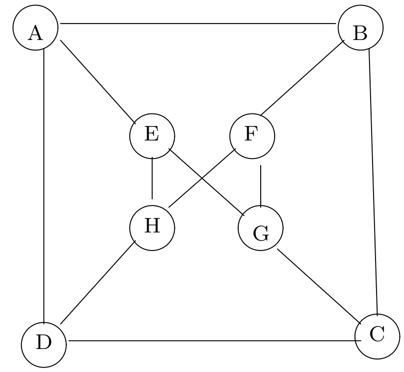
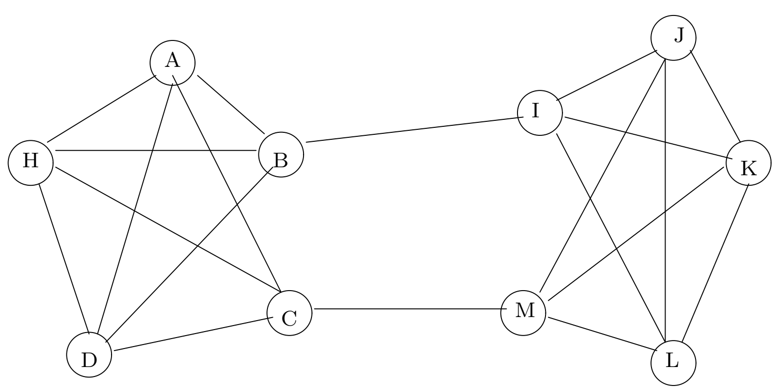
1. (13 points) Are the two graphs shown below isomorphic? If so, give the isomorphism; if not, give careful reasons for your answer.




2. (12 points) Compute the chromatic number (vertex colouring number) of the graph $G$ shown below. Justify your answer! (Show a colouring with $\chi(G)$ colors (label each node with its colour), and argue that fewer colours cannot suffice.)



3. (25 points) True or False? If true, give a short proof. If false, give a counterexample:

(a). Every subgraph of a bipartite graph is also bipartite.

(b). Every subgraph of a complete graph is also a complete graph.

(c). Let $H$ be a subgraph of $G$, then $\chi(H) \leq \chi(G)$.

(d). If $G$ contains a $K_5$ configuration then $\chi(G) \geq 5$.

(e). Let $G$ be a graph that has an Euler cycle, then $G$ must also have a Hamilton circuit.

4. (12 points) Model the following problem as a graph colouring problem: Stony Brook students are trying to schedule meetings of 12 student committees. Each meeting is to be held during Wednesday's campus life time 12:50-2:10. Several students are very active and are part of more than one committee. The schedule should be arranged so that such students can attend all meetings of committees to which they belong.

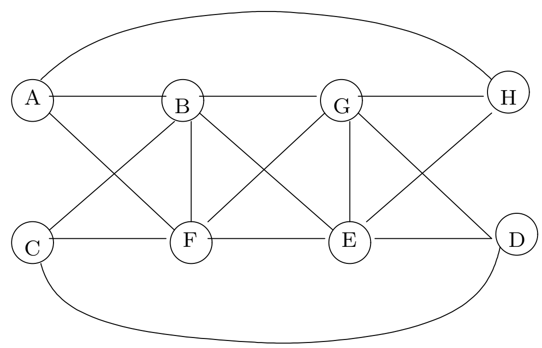
(a). What exactly does the set of vertices $V$ correspond to?

(b). What exactly does the set of edges $E$ correspond to?

(c). What is being coloured, the vertices or the edges?

(d). You are told that the graph you defined can be coloured with 3 colours. Can all committee meetings be scheduled within the month of February, without conflicts? Explain!

5. (13 points) Show that the following graph is non planar by showing the $K_{3,3}$ or $K_5$ configuration it contains.




6. (12 points) A planar connected graph $G$ has 10 nodes each of degree 4.

(a). How many edges does the graph $G$ have?

(b). How many regions does the graph $G$ have?

(c). Is it possible that $G$ is bipartite? Explain!

7. (13 points) Prove that no Hamilton circuit exists in the following graph: